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HOT-WIRE METHOD IN A NONSTATIONARY VARIATION

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Starting from the solution of the thermal-conductivity equation, a nonstationary variation of the "hot-wire" method is developed in the case of monotonic heating of a calorimetric system which permits determination of the temperature dependence of the thermal conductivity of liquids in a broad temperature range from one test.

The stationary "hot-wire" method is extensively used at this time to investigate the coefficient of thermal conductivity of gases and liquids [1]. This method, as all stationary methods (plane layer, coaxial cylinders), is distinctive in the long duration of the test, requires a complex apparatus, and does not permit determination of the temperature dependence of the coefficient of thermal conductivity from one test. Using this method, the experimenter should expect the buildup of a stationary state in a calorimetric system every time when measuring the coefficient of thermal conductivity. Consequently, the determination of the coefficient of thermal conductivity of one liquid in a broad temperature range takes several days at a minimum. Hence, several, principally foreign, papers devoted to a nonstationary variation of the "hot-wire" method have recently appeared [2-15]. The theory of the method in application to rarefied gases is elucidated in especial detail in [16]. The "hot-wire" method differs from all other nonstationary methods in that the coefficient of thermal conductivity is determined directly by this method, and not the coefficient of thermal diffusivity. However, it is not very exact because of the difficulty of recording exactly the rapidly varying wire temperature during the measurement. In this respect, the method mentioned in the relative variation in which the recording device acts as a zero indicator [17] is of definite interest.

An attempt is made below to extend the "hot-wire" method to the case of a monotonic change in the temperature of a calorimetric system.

The design scheme of the method under consideration reduces to the following. A fine metal wire of radius R_i (Fig. 1) is stretched coaxially in a bulky metal tube 1 of radius R_0 through a sealed electrical insulating plug 2. The liquid being investigated fills the gap between the wire 3 and the tube 1. A constant-power electrical current passes through the wire during the entire test. In the stationary variation of the method, the whole system (module) is strictly thermostated.

Let us assume that the whole system is surrounded by a heat-insulating shell which rises smoothly in temperature under the effect of the external heater 4 in such a way that the temperature of the shell approximately equals the temperature of the module. In this case the heat flux of the inner heater 3 is expended completely in a slow rise in the temperature of the module and the liquid. The power $W(\tau)$ of the Lenz–Joule heat developed by the wire, the temperature drop $\vartheta(\tau)$ in the layer under investigation, and the rate of temperature rise $b(\tau)$ of the system are measured in the test.

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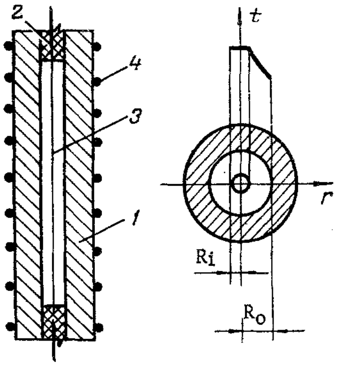

Fig. 1. Calorimeter diagram, in principle, and temperature distribution therein: 1) metal tube; 2) plug; 3) platinum wire; 4) heater.

The geometric parameters of the calorimetric system are assumed known and the temperature drop $\vartheta(\tau)$ is small.

The theory of such a system reduces to the following. The temperature field of a cylindrical layer is described by the thermal-conductivity equation

$$\frac{\partial^2 t}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial t}{\partial r}=\frac{1}{a}\cdot\frac{\partial t}{\partial \tau}. \tag{1}$$

If the temperature drop $\vartheta(r, \tau) = t(r, \tau) - t(R_o, \tau)$ is introduced, then in place of (1) we have

$$\frac{\partial^2 \vartheta}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial \vartheta}{\partial r}=\frac{b}{a}. \tag{2}$$

Let us represent (2) in the form

$$\frac{\partial}{\partial r}\left(r\frac{\partial \vartheta}{\partial r}\right)=\frac{b}{a}r. \tag{3}$$

Integrating (3), we obtain

$$\vartheta(r, \tau)=\frac{b}{4a}r^2+A\ln r+B, \tag{4}$$

where A and B are constants of integration determined from the following boundary conditions:

$$\begin{gathered}\vartheta(r, \tau)|_{r=R_o}=0,\\ \vartheta(r, \tau)|_{r=R_i}=\vartheta_{i-o},\\ \lambda\frac{\partial\vartheta(r, \tau)}{\partial r}\Big|_{r=R_i}=-\frac{W}{F_i}\end{gathered} \tag{5}$$

From conditions (5) we obtain

$$B=-\frac{b}{4a}R_o^2-A\ln R_o, \tag{6}$$

$$\vartheta_{i-o}=A\ln\frac{R_i}{R_o}-\frac{b}{4a}(R_o^2-R_i^2), \tag{7}$$

$$A=\frac{W}{2\pi l\lambda}-\frac{b}{2a}R_i^2. \tag{8}$$

After substituting (8) into (7), we will have

$$\vartheta_{i-o}=\frac{W}{2\pi l\lambda}\ln\frac{R_o}{R_i}-\frac{b}{4a}\left[R_o^2-R_i^2\left(1+2\ln\frac{R_o}{R_i}\right)\right]. \tag{9}$$

The first member in the right side of this last equation determines the stationary component and the second member, the nonstationary component, of the drop ϑ_{i-o}. It is hence important that the second member in the right side of (9) be small compared to the first in the test.

From (9) we will have

$$\lambda = \frac{W}{2\pi l\vartheta_{i-o}} \ln \frac{R_o}{R_i} - \frac{1}{4}\gamma cb \frac{1}{\vartheta_{i-o}}\left[R_o^2 - R_i^2\left(1 + 2\ln\frac{R_o}{R_i}\right)\right]. \tag{10}$$

If it is taken into account that part of the heat liberated by the wire is absorbed, then $W = W_o - C_o b$. Taking account of the heat absorbed by the wire, the computation formula becomes

$$\lambda(t) = \frac{W_o \ln \dfrac{R_o}{R_i}}{2\pi l\vartheta_{i-o}}(1 - \Delta\sigma_c), \tag{11}$$

where

$$\Delta\sigma_c = \frac{cb}{2W_o}\left(\frac{1}{\ln\dfrac{R_o}{R_i}} - \frac{2R_i^2}{R_o^2 - R_i^2}\right) + \frac{C_o b}{W_o}. \tag{12}$$

It follows from the computation formula (11) obtained that to determine λ(t) in the method under consideration it is sufficient to know the temperature dependences $W_o(t)$ and b(t) in the heating section. Measurements of the power $W_o(t)$ of the electrical source can be accomplished by the usual means – by recording the current I(t) and voltage drop $U_l(t)$ on the working section of the wire. It is convenient to represent the correction $\Delta\sigma_c$ in the form

$$\Delta\sigma_c = \frac{1}{4}\cdot\frac{c\gamma b}{\lambda\vartheta_{i-o}}\left\{\left[R_o^2 - R_i^2\left(1 + 2\ln\frac{R_o}{R_i}\right)\right] + 2\frac{c_o\gamma_o}{c\gamma}R_i^2\ln\frac{R_o}{R_i}\right\}. \tag{13}$$

Such a form of the correction is convenient for computations in the design stage of the experimental apparatus. To estimate the optimal heating rate, the dependence

$$b_{adm} = \frac{4\lambda\vartheta_{i-o}\Delta\sigma_c}{c\gamma\left[R_o^2 - R_i^2\left(1 + 2\ln\dfrac{R_o}{R_i}\right)\right] + 2c_o\gamma_o R_i^2 \ln\dfrac{R_o}{R_i}} \tag{14}$$

resulting from (13) can be used. As is seen from (13), the correction $\Delta\sigma_c$ has a clear structure, contains sufficiently definite parameters, and admits of an analytical estimation, all the more since its magnitude does not usually exceed 1%.

It is expedient to use the relationships (13) and (14) in the design stage of the experimental apparatus in order to assure a reduction in the magnitude of the correction $\Delta\sigma_c$ to negligibly small values. The initial data for the thermal computation of the calorimetric system in the design stage are the expected values of the thermal conductivity of the substances to be investigated, the admissible temperature drop in the layer ϑ_{i-o}, and the geometric parameters of the system R_o, R_i.

Let us solve a numerical example on the basis of the obtained computation formula. Let the calorimetric apparatus be intended to investigate the coefficient of thermal conductivity of nonmetallic liquids with a thermal conductivity of $\lambda \leq 0.2$ W/m·deg. The geometric parameters of the calorimetric system are as follows: $R_i = 0.05$ mm and $R_o = 0.4$ mm. The temperature drop in the layer is maintained at $\vartheta_{i-o} = 10$ deg.

Taking $c\gamma = 2\cdot10^6$ J/m^3·deg for organic fluids, $c_o\gamma_o = 2.8\cdot10^6$ J/m^3·deg for the platinum wire, and starting from (14), for the admissible magnitude of the heating rate we obtain $b_{adm} \simeq 25\,\Delta\sigma_c$.

The optimal values of the heating rate ordinarily lie within the limits b = (0.1-0.3) deg/sec. Let us take b = 0.1 deg/sec. In this case the correction is $\Delta\sigma_c = 0.4\cdot10^{-2} = 0.4\%$. For $\Delta\sigma_c = 0.01$ the admissible heating rate is 0.25 deg/sec.

As is seen from the computation presented, measurements in the nonstationary mode are completely admissible in the hot-wire method. If the cγ of the substance grows even 10-fold near the critical domain, then even in this case a test in the nonstationary mode can be set up and the correction $\Delta\sigma_c$ can be maintained negligibly small. Hence, all the ordinary corrections inherent in the stationary variation of the method (heat efflux through the end faces, readings on the temperature sensors, radiation, etc.) remain valid, since the test mode remains as before, in practice.* Hence, it can be hoped that the experimental error in metrological respects will be as before (2-2.5%) without any noticeable degradation in accuracy. It should be noted that the electrical

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*The method of taking these corrections into account is elucidated in detail in [1], for example.

resistivity of the wire will change with the rise in temperature, which will result in a change in heat liberation in the wire during the measurement. This effect can be computed by the method in [4, 5] and [15]. However, the effect mentioned can be cancelled completely by experimental means also for a low heating rate [14].

If the test duration to obtain the temperature dependence $\lambda(t)$ in a temperature range between room temperature and 400°C is a minimum of several days in the stationary mode, then this dependence can be obtained in the stream $\tau = \Delta t | b = 380/0.1 \simeq 63$ min in the dynamic mode with a $b = 0.1$ deg/sec heating rate.

NOTATION

R_i and R_o	are the wire and tube radii, respectively;
$b = dt/d\tau$	is the heating rate of system;
$\vartheta(\tau)$	is the temperature drop;
$W(\tau)$	is the power;
λ, a	are the coefficients of thermal conductivity and thermal diffusivity, respectively;
C, C_o	are the specific heats of the fluid and wire, respectively;
γ, c	are the fluid density and specific heat;
γ_o and c_o	are the density and specific heat of the platinum wire;
$\Delta\sigma_c$	is the correction to the specific heat of the layer.

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